UNIT - II

INTRODUCTION

The importance of linear multivariable control systems is evidenced by the papers published in recent years. Despite the extensive literature certain fundamental matters are not well understood. This is confirmed by numerous inaccurate stability analyses, erroneous statements about the existence of stable control, and overly severe constraints on compensator characteristics. The basic difficulty has been a failure to account properly for all dynamic modes of system response. This failure is attributable to a limitation of the transfer-function matrix-- it fully describes a linear system if and only if the system is controllable and observable. The concepts of controllability and observability were introduced by Kalman and have been employed primarily in the study of optimal control. In this paper, the primary objective is to determine the controllability and observability of composite systems which are formed by the interconnection of several multivariable subsystems. To avoid the limitations of the transfer-function matrix, the beginning sections deal with multivariable systems as described by a set of n first order, constant coefficient differential equations. Later, the extension to systems described by transfer-function matrices is made. Throughout, emphasis is on the fundamental aspects of describing multivariable control systems. Detail design procedures are not treated.
Introduction In the context of this course, the main objective of using state-space equations to model systems is the design of suitable compensation schemes to control these systems. Typically, the control signal u(t) is a function of several measurable state variables. Thus, a state variable controller that operates on the measurable information is developed. State variable controller design is typically comprised of three steps: Assume that all the state variables are measurable and use them to design a full state feedback control law. In practice, only certain states or combination of them can be measured and provided as system outputs. An observer is constructed to estimate the states that are not directly sensed and available as outputs. Reduced-order observers take advantage of the fact that certain states are already available as outputs and they don't need to be estimated. Appropriately connecting the observer to the full-state feedback control law yields a state-variable controller, or compensator.

Definitions and notation:

Controllability and Controllability matrix Definition: A control system is said to be (completely) controllable if, for all initial times t0 and all initial states x(t0), there exists some input function u(t) that drives the state vector x(t) to any final state at some finite time t0<t<T

CONTROLLABILITY TEST: Given a system defined by the linear state equation the controllability matrix is defined as: It can be proved that a system is controllable if and only if: For the general multiple-input (m) case, A is an n x n matrix and B is n x m. Then, P consists of n matrix blocks[B, AB, A2B …An-1B,]each with dimension n x m, stacked side by side. Thus, P has dimension n x n·m, having more columns than rows. For the single-input case, B consists of a single column, yielding a square n x n controllability matrix P. Therefore, a single-input linear system is controllable if and only if the associated controllability matrix P is nonsingular.

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathrm{u}$$

$$\mathrm{rank}[\mathbf{P}_c] = n$$

$$|\mathbf{P}_c| \neq 0$$

$$\text{Controllability Matrix } \mathbf{P}_c = [\mathbf{B}\ \mathbf{AB}\ \mathbf{A}^2\mathbf{B}\ \cdots \mathbf{A}^{n-1}\mathbf{B}]$$

Observability

In state-space description of linear time-invariant systems, the state vector x(t) is an internal quantity that is influenced by the input u(t) and affects the output y(t). In practice, the dimension of x(t) is greater than the number of inputs or output signals

The question stated now is whether or not the initial state can be uniquely determined by measurements of the input and output signals of the linear s-s equation over a finite time interval. If so, initial state and input signal can be injected to the state equation solution formula to reconstruct (predict) the entire state trajectory.

Definition: Given a LTI system that is described by the state x(t0) is said to be observable if given any input u(t), there exists a finite time T >t0 such that the knowledge of signal input u(t), signal output y(t) for T > t t0, and matrices A, B, C and D, are sufficient to determine x(t0). If every state of the system is observable, the system is said to be (completely) observable.

Observability matrix OBSERVABILITY TEST :

Given a system defined by its linear state equation, the observability matrix is defined as: It can be proved that a system is observable if and only if: For the general multiple-output (p) case, A is an n x n matrix and C is p x n. Then, Q consists of n matrix blocks [C, CA, CA2 …CAn-1], each with dimension p x n, stacked one on top of another. Thus, Q has dimension np x n, having more rows than columns. For the single-output case, C consists of a single row, yielding a square n x n observability matrix Q. Therefore, a single-output linear system is observable if and only if the associated observability matrix Q is nonsingular. (|Q | not equal to0)

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

$$y = \mathbf{C}\mathbf{x}$$

$$\text{rank}[\mathbf{P}_c] = n$$

$$\text{Observability Matrix } \mathbf{P}_o = [\mathbf{C}\,\mathbf{C}\mathbf{A} \cdots \mathbf{C}\mathbf{A}^{n-1}]^T$$

IMPORTANCE

The concept of controllability and observability play a vital role in the design of control systems in state space. They govern the existence of a complete solution to the control system design problem, the solution to this problem may not exist if the system considered is not controllable.

It is important to note that all physical systems are controllable and observable. however the mathematical models of these system may not possess the property of the controllability or observability, then it is necessary to know the conditions under which a system is controllable and observable and the designer can seek another state model which is controllable and observable.

The controllability test is necessary to find the usefulness of a state variable. if the state variables are controllable by controlling (i.e. varying)the state variables the desired outputs of the system is achieved.

The observability test is necessary to find the whether the state variables are measurable or not. if the state variables are measurable then the state of the system can be determined by practical measurements of the state variables.

APPLICATIONS

Controllability is an important property of a control system, and the controllability property plays a crucial role in many control problems, such as stabilization of unstable systems by **feedback, or optimal control**.

Controllability and observability are **dual aspects of the same problem.**

Roughly, the concept of controllability denotes the ability to move a system around in its entire configuration space using only certain admissible manipulations. The exact definition varies slightly within the framework or the type of models applied.