Chapter 8

Acoustics

This chapter outlines the brief introduction to acoustics given in class in somewhat more detail.

8.1 Formulation

We reduce the Euler equations for isentropic flow to the following equations where quantities with a hat are understood to be small perturbations about the ambient state, denoted with a subscript of "o".

$$\frac{\partial \hat{\rho}}{\partial t} + \rho_o \nabla \cdot \hat{\mathbf{v}} = 0 \tag{8.1}$$

$$\rho_o \frac{\partial \hat{\mathbf{v}}}{\partial t} + \nabla \cdot \hat{P} = 0 \tag{8.2}$$

$$\hat{P} = c_o^2 \hat{\rho}. \tag{8.3}$$

Introducing the velocity potential $\nabla\phi = \hat{\mathbf{v}}$ and employing further manipulation allows the equation to be written as the wave equation:

$$\frac{\partial^2 \phi}{\partial t^2} = c_o^2 \nabla^2 \phi. \tag{8.4}$$

The pressure, velocity, and density are then obtained from

$$\hat{P} = -\rho_o \frac{\partial \phi}{\partial t}, \tag{8.5}$$

$$\hat{\mathbf{v}} = \nabla \phi, \tag{8.6}$$

$$\hat{\rho} = -\rho_o c_o^2 \frac{\partial \phi}{\partial t}. \tag{8.7}$$

8.2 Planar Waves

The D'Alembert solution for planar waves is shown in class to be

$$\phi = f(x + c_o t) + g(x - c_o t), \tag{8.8}$$
$$\hat{P} = -\rho_o c_o f'(x + c_o t) + \rho_o c_o g'(x - c_o t), \tag{8.9}$$
$$\hat{u} = f'(x + c_o t) + g'(x - c_o t), \tag{8.10}$$
$$\tag{8.11}$$

8.3 Spherical Waves

The D'Alembert solution for spherical waves is shown in class to be

$$\phi = \frac{1}{r} f(r + c_o t) + \frac{1}{r} g(r - c_o t), \tag{8.12}$$
$$\hat{P} = -\frac{\rho_o c_o}{r} f'(r + c_o t) + \frac{\rho_o c_o}{r} g'(r - c_o t), \tag{8.13}$$
$$\hat{u} = \frac{1}{r} f'(r + c_o t) + \frac{1}{r} g'(r - c_o t), \tag{8.14}$$
$$\tag{8.15}$$