LECTURE NOTES

ON

NETWORK THEORY

II B. Tech II semester (JNTUH-R15)

ELECTRICAL AND ELECTRONICS ENGINEERING

Unit 1
Three Phase AC Circuits

Introduction

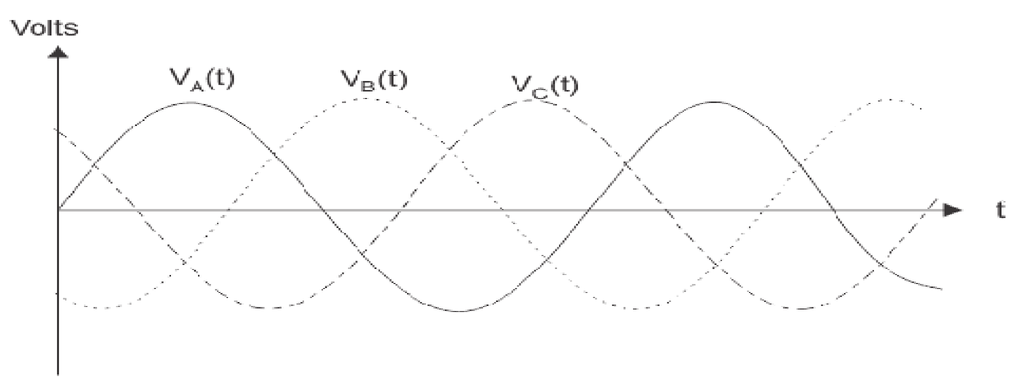
Three-phase systems are commonly used in generation, transmission and distribution of electric power. Power in a three-phase system is constant rather than pulsating and three-phase motors start and run much better than single-phase motors. A three-phase system is a generator-load pair in which the generator produces three sinusoidal voltages of equal amplitude and frequency but differing in phase by 120° from each other.

There are two types of system available in electric circuit, single phase and **three phase system**. In single phase circuit, there will be only one phase, i.e the current will flow through only one wire and there will be one return path called neutral line to complete the circuit. So in single phase minimum amount of power can be transported. Here the generating station and load station will also be single phase. This is an old system using from previous time.

In polyphase system, that more than one phase can be used for generating, transmitting and for load system. **Three phase circuit** is the polyphase system where three phases are send together from the generator to the load. Each phase are having a phase difference of 120°, i.e 120° angle electrically. So from the total of 360°, three phases are equally divided into 120° each. The power in **three phase system** is continuous as all the three phases are involved in generating the total power. The sinusoidal waves for 3 phase system is shown below The three phases can be used as single phase each. So if the load is single phase, then one phase can be taken from the **three phase circuit** and the neutral can be used as ground to complete the circuit..

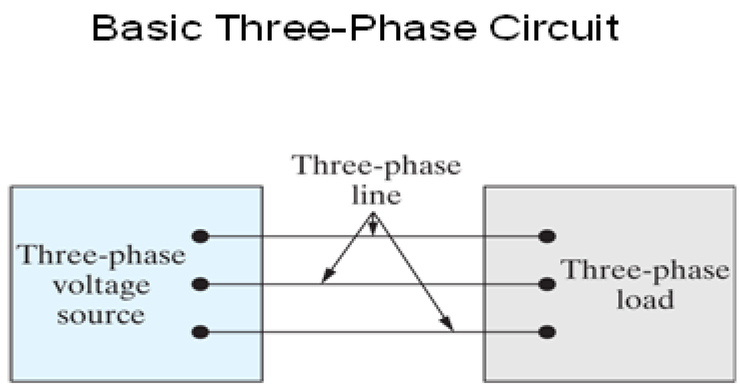
The phase voltages $v_a(t)$, $v_b(t)$ and $v_c(t)$ are as follows

$$
\begin{aligned}
v_a &= V_m \cos \omega t \\
v_b &= V_m \cos\left(\omega t - 120^\circ\right) \\
v_c &= V_m \cos\left(\omega t - 240^\circ\right),
\end{aligned}
$$



the corresponding phasors are

$$
\begin{aligned}
V_a &= V_m \\
V_b &= V_m e^{-j120^\circ} \\
V_c &= V_m e^{-j240^\circ} \ .
\end{aligned}
$$



Basic three Phase Circuit

Advantages of Three Phase is preferred Over Single Phase

The three phase system can be used as three single phase line so it can act as three single phase system. The three phase generation and single phase generation is same in the generator except the arrangement of coil in the generator to get 120° phase difference. The conductor needed in three phase circuit is 75% that of conductor needed in single phase circuit. And also the instantaneous power in single phase system falls down to zero as in single phase we can see from the sinusoidal curve but in three phase system the net power from all the phases gives a continuous power to the load. the will have better and higher efficiency compared to the single phase system.

In three phase circuit, connections can be given in two types:

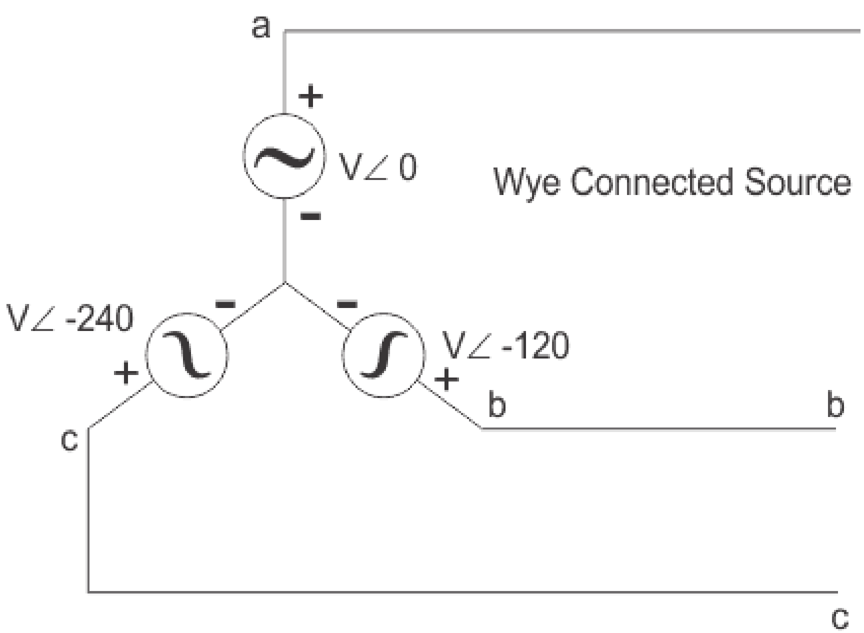
1. Star connection
2. Delta connection

STAR CONNECTION

In star connection, there is four wire, three wires are phase wire and fourth is neutral which is taken from the star point. Star connection is preferred for long distance power transmission because it is having the neutral point. In this we need to come to the concept of balanced and unbalanced current in power system.

When equal current will flow through all the three phases, then it is called as balanced current. And when the current will not be equal in any of the phase, then it is unbalanced current. In this case, during balanced condition there will be no current flowing through the neutral line and hence there is no use of the neutral terminal. But when there will be unbalanced current flowing in the three phase circuit, neutral is having a vital role. It will take the unbalanced current through to the ground and protect the transformer. Unbalanced current affects transformer and it may also cause damage to the transformer and for this star connection is preferred for long distance transmission.

THE STAR CONNECTION

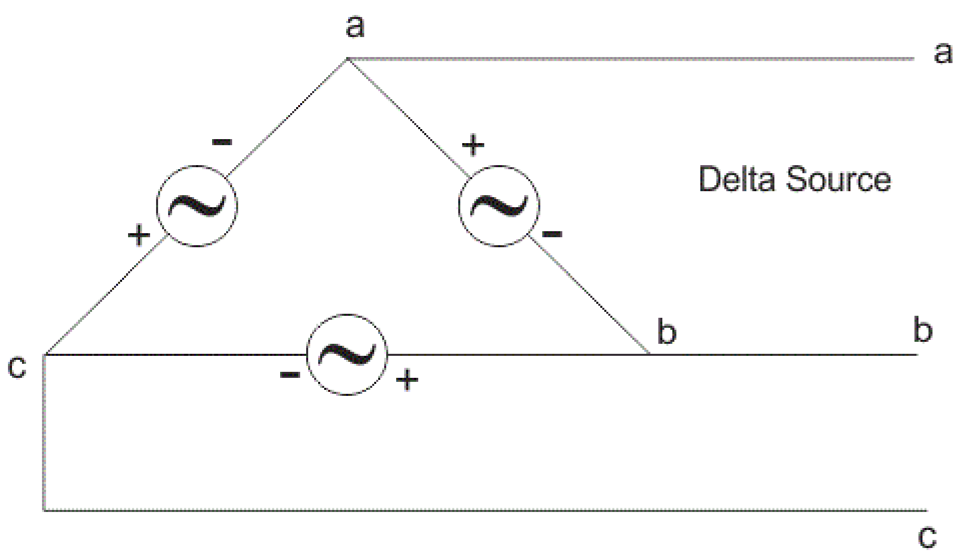


In star connection, the line voltage is √3 times of phase voltage. Line voltage is the voltage between two phases in three phase circuit and phase voltage is the voltage between one phase to the neutral line. And the current is same for both line and phase. It is shown as expression below

$$E_{Line} = \sqrt{3}E_{phase} \; and \; I_{Line} = I_{Phase}$$

Delta Connection

In delta connection, there is three wires alone and no neutral terminal is taken. Normally delta connection is preferred for short distance due to the problem of unbalanced current in the circuit. The figure is shown below for delta connection. In the load station, ground can be used as neutral path if required.



In delta connection, the line voltage is same with that of phase voltage. And the line current is √3 times of phase current. It is shown as expression below,

$$E_{Line} = E_{phase} \; and \; I_{Line} = \sqrt{3}I_{Phase}$$

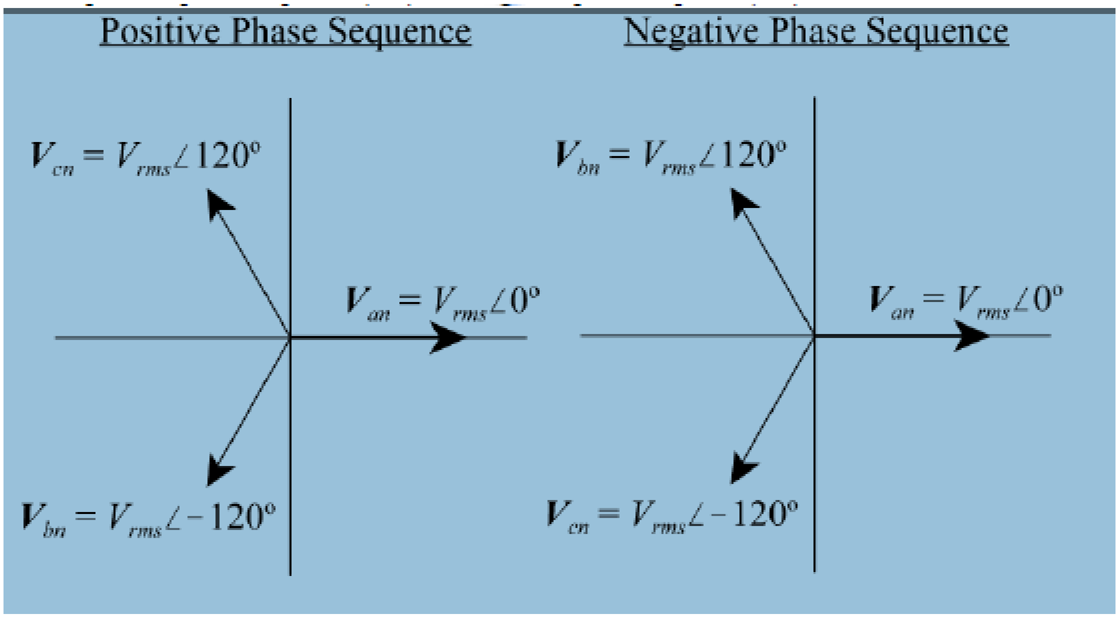
If we compare the line-to-neutral voltages with the line-to-line voltages, we find the following relationships,

Line-to-neutral voltages	Line-to-line voltages
$V_{an} = V_{rms}\angle 0^\circ$	$V_{ab} = \sqrt{3}V_{rms}\angle 30^\circ$
$V_{bn} = V_{rms}\angle -120^\circ$	$V_{bc} = \sqrt{3}V_{rms}\angle -90^\circ$
$V_{cn} = V_{rms}\angle 120^\circ$	$V_{ca} = \sqrt{3}V_{rms}\angle 150^\circ$

In three phase circuit, star and delta connection can be arranged in four different ways-

1. Star-Star connection
2. Star-Delta connection
3. Delta-Star connection
4. Delta-Delta connection

Phase Sequence



But the power is independent of the circuit arrangement of the three phase system. The net power in the circuit will be same in both star and delta connection. The power in three phase circuit can be calculated from the equation below,

$$P_{Total} = 3 \times E_{phase} \times I_{phase} \times PF$$

Since there is three phases, so the multiple of 3 is made in the normal power equation and the PF is power factor. Power factor is a very important factor in three phase system and some times due to certain error, it is corrected by using capacitors.

ANALYSIS OF BALANCED THREE PHASE CIRCUITS

In a balanced system, each of the three instantaneous voltages has equal amplitudes, but is separated from the other voltages by a phase angle of 120. The three voltages (or phases) are typically labeled a, b and c. The common reference point for the three phase voltages is designated as the neutral connection and is labeled as n.

A three-phase system is shown in Fig 1. In a special case all impedances are identical $Z_a = Z_b = Z_c = Z$

Such a load is called a balanced load and is described by equations

$$I_a = \frac{V_a}{Z} \qquad I_b = \frac{V_b}{Z} \qquad I_c = \frac{V_c}{Z}\ .$$

Using KCL, we have

$$I_n = I_a + I_b + I_c = \frac{1}{Z}\left(V_a + V_b + V_c\right) \ ,(4)$$

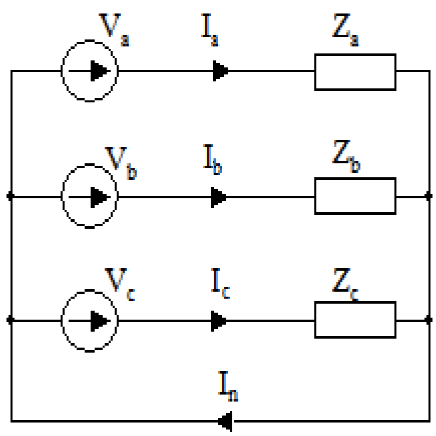


Fig. 1

$$V_a + V_b + V_c = V_m\left(1 + e^{-j120^\circ} + e^{-j240^\circ}\right) =$$
$$= V_m\left(1 + \cos 120^\circ - j\sin 120^\circ + \cos 240^\circ - j\sin 240^\circ\right) = V_m\left(1 - \frac{1}{2} - j\frac{\sqrt{3}}{2} - \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = 0\ .$$

From the above result, we obtain $I_n = 0$.

Since the current flowing though the fourth wire is zero, the wire can be removed

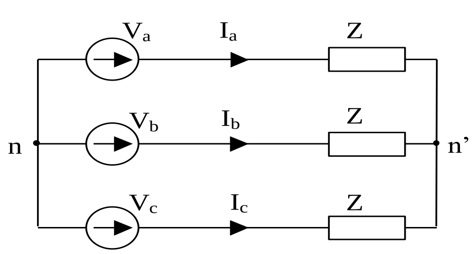
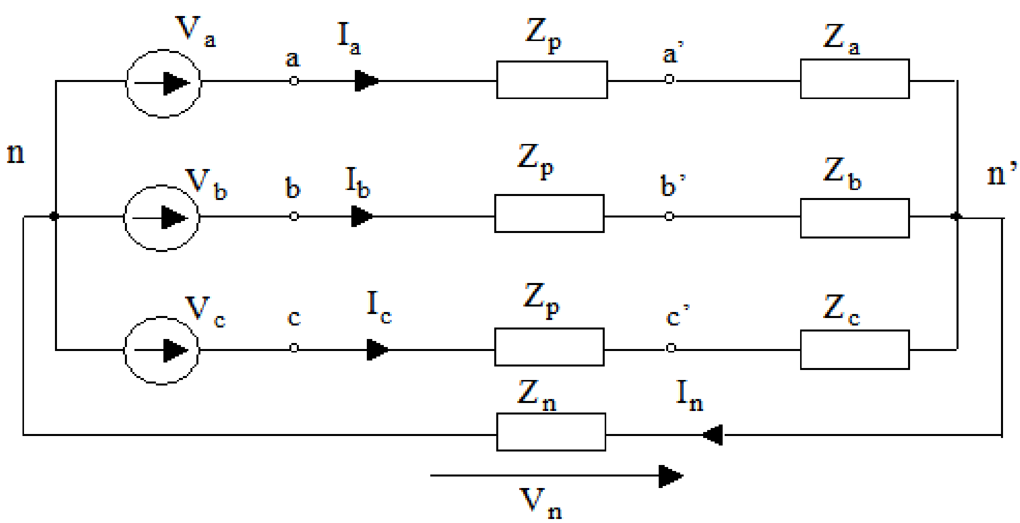


Fig. 2

ANALYSIS OF UNBALANCED LOADS

Three-phase systems deliver power in enormous amounts to single-phase loads such as lamps, heaters, air-conditioners, and small motors. It is the responsibility of the power systems engineer to distribute these loads equally among the three-phases to maintain the demand for power fairly balanced at all times. While good balance can be achieved on large power systems, individual loads on smaller systems are generally unbalanced and must be analyzed as unbalanced three phase systems.

When the three phases of the load are not identical, an unbalanced system is produced. An unbalanced Y-connected system is shown in Fig.1. The system of Fig.1 contains perfectly conducting wires connecting the source to the load. Now we consider a more realistic case where the wires are represented by impedances Z_p and the neutral wire connecting n and n' is represented by impedance Z_n



the node n as the datum, we express the currents I_a, I_b, I_c and I_n in terms of the node voltage V_n

$$I_a = \frac{V_a - V_n}{Z_a + Z_p}$$

$$I_b = \frac{V_b - V_n}{Z_b + Z_p}$$

$$I_c = \frac{V_c - V_n}{Z_c + Z_p}$$

$$I_n = \frac{V_n}{Z_n}$$

The node equation is

$$\frac{V_n}{Z_n} - \frac{V_a - V_n}{Z_a + Z_p} - \frac{V_b - V_n}{Z_b + Z_p} - \frac{V_c - V_n}{Z_c + Z_p} = 0$$

And $$V_n = \frac{\dfrac{V_a}{Z_a + Z_p} + \dfrac{V_b}{Z_b + Z_p} + \dfrac{V_c}{Z_c + Z_p}}{\dfrac{1}{Z_n} + \dfrac{1}{Z_a + Z_p} + \dfrac{1}{Z_b + Z_p} + \dfrac{1}{Z_c + Z_p}}$$

Power in three-phase circuits

In the balanced systems, the average power consumed by each load branch is the same and given by

$$\tilde{P}_{av} = V_{eff} I_{eff} \cos\phi$$

where V_{eff} is the effective value of the phase voltage, I_{eff} is the effective value of the phase current and ϕ is the angle of the impedance. The total average power consumed by the load is the sum of those consumed by each branch, hence, we have

$$P_{av} = 3\tilde{P}_{av} = 3V_{eff} I_{eff} \cos\phi$$

In the balanced Y systems, the phase current has the same amplitude as the line current $I_{eff} = (I_{eff})_L$, whereas the line voltage has the effective value $(V_{eff})_L$ which is $\sqrt{3}$ times greater than the effective value of the phase voltage, $(V_{eff})_L = \sqrt{3}V_{eff}$. Hence, using (22), we obtain

$$P_{av} = 3\frac{(V_{eff})_L}{\sqrt{3}}(I_{eff})_L \cos\phi = \sqrt{3}(V_{eff})_L (I_{eff})_L \cos\phi$$

Similarly, we derive

$$P_x = \sqrt{3}(V_{eff})_L (I_{eff})_L \sin\phi$$

In the unbalanced systems, we add the powers of each phase

$$P_{av} = (V_{eff})_a (I_{eff})_a \cos\phi_a + (V_{eff})_b (I_{eff})_b \cos\phi_b + (V_{eff})_c (I_{eff})_c \cos\phi_c$$

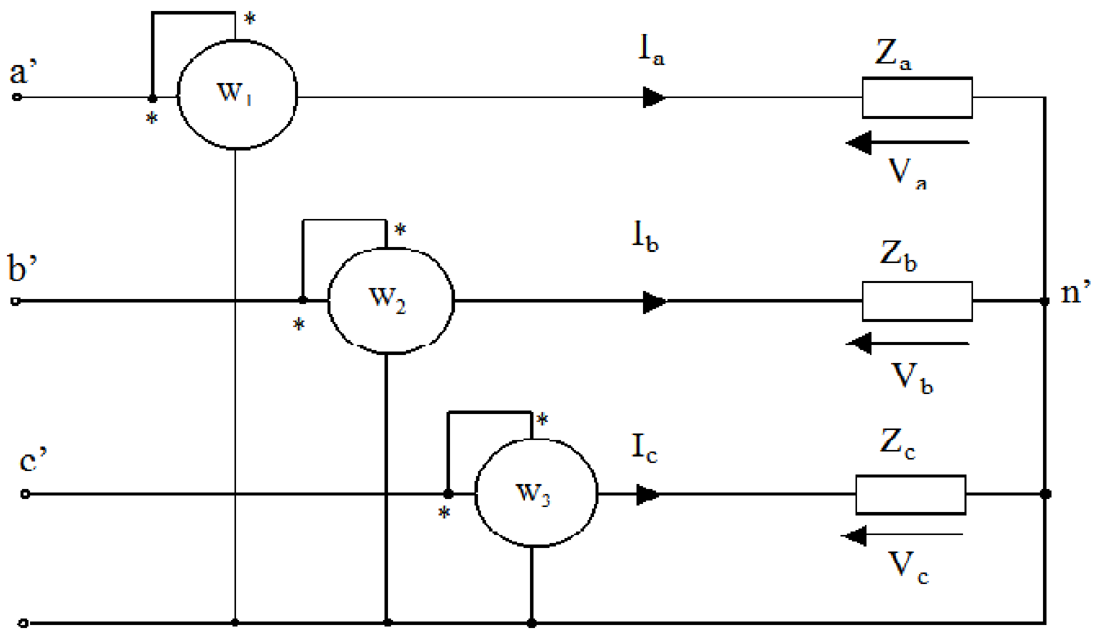
$$P_x = (V_{eff})_a (I_{eff})_a \sin\phi_a + (V_{eff})_b (I_{eff})_b \sin\phi_b + (V_{eff})_c (I_{eff})_c \sin\phi_c$$

Three wattmeter method

In order to measure the average power in a three-phase Y-connected load, we use three wattmeters connected as shown.

The reading of the wattmeter W_1 is

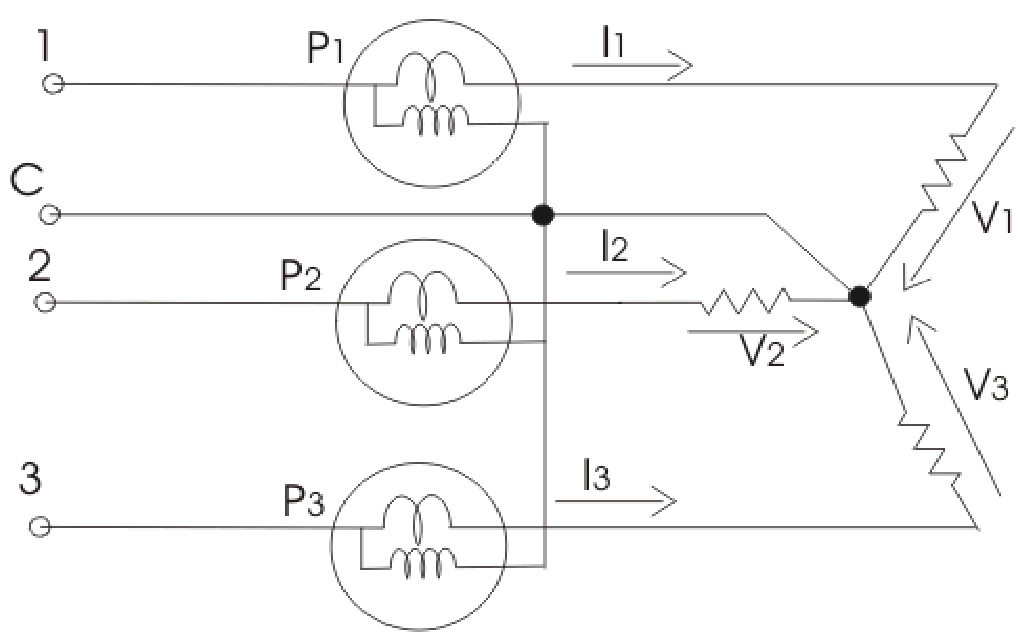
$$P_{W_1} = \frac{1}{2}\mathrm{Re}\left(V_a I_a^*\right) = \frac{1}{2}(V_m)_a (I_m)_a \cos\phi_a = (V_{eff})_a (I_{eff})_a \cos\phi_a = P_a$$



Similarly, W_2 and W_3 measure the average power of the load branch b and c, respectively. Thus, the sum of the three readings will give the total average power. This method of the average power measurement is valid for both balanced and unbalanced Y-connected loads. Note that in the case of a balanced Y-connected load all three readings are identical and therefore we use only one wattmeter

Measurement of Three Phase Power by Three Wattcmeters Method

The circuit diagram is shown below-

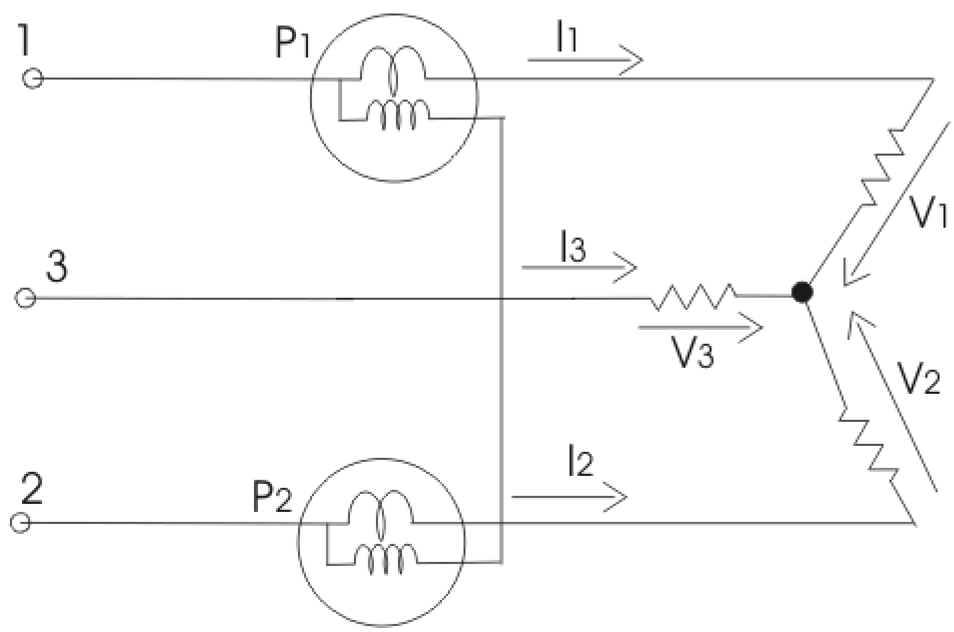


Here, it is applied to three phase four wire systems, current coil of all the three wattmeters marked as one, two and three are connected to respective phases marked as one, two and three. Pressure coils of all the three wattmeter are connected to common point at neutral line. Clearly each wattmeter will give reading as product of phase current and line voltage which is phase power. The resultant sum of all the readings of wattmeter will give the total power of the circuit. Mathematically we can

$$P = P_1 + P_2 + P_3 = V_1 I_1 + V_2 I_2 + V_3 I_3$$

Measurement of Three Phase Power by Two Wattmeters Method

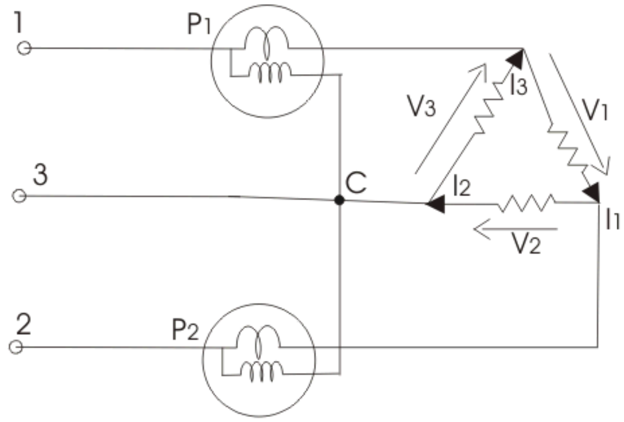
In this method we have two types of connections
(a) Star connection of loads
(b) Delta connection of loads.
When the star connected load, the diagram is shown in below-



For star connected load clearly the reading of wattmeter one is product phase current and voltage difference (V_2-V_3). Similarly the reading of wattmeter two is the product of phase current and the voltage difference (V_2-V_3). Thus the total power of the circuit is sum of the reading of both the wattmeters. Mathematically we can write

$$P = P_1 + + P_2 = I_1(V_1 + V_2) + I_2(V_2 - V_3)$$

but we have $I_1+I_2+I_3=0$,hence putting the value of $I_1+I_2=-I_3$. We get total power as $V_1I_1+V_2I_2+V_3I_3$.
For delta connected load, the diagram is shown in below



The reading of wattmeter one can be written as

$$P_1 = -V_3(I_1 - I_3)$$

and reading of wattmeter two is

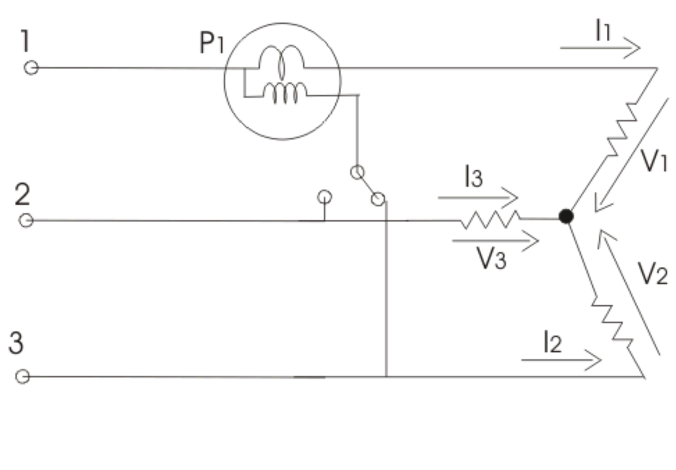
$$P_2 = -V_2(I_2 - I_1)$$

$$Total\ power\ is\ \ P = P_1 + P_2 = V_2 I_2 + V_3 I_3 - I_1(V_2 + V_3)$$

but $V_1+V_2+V_3=0$, hence expression for total power will reduce to $V_1I1+V_2I_2+V_3I_3$.

Measurement of Three Phase Power by One Wattmeter Method

Limitation of this method is that it cannot be applied on unbalanced load. So under this condition we have $I_1=I_2=I_3=I$ and $V_1=V_2=V_3=V$.
Diagram is shown below:



Two switches are given which are marked as 1-3 and 1-2, by closing the switch 1-3 we get reading of wattmeter as

$$P_1 = V_{13} I_1 \cos(30 - \phi) = \sqrt{3} \times VI\cos(30 - \phi)$$

Similarly the reading of wattmeter when switch 1-2 is closed is

$$P_2 = V_{12} I_1 \cos(30 + \phi) = \sqrt{3} \times VI \cos(30 + \phi)$$

$$Total\ power\ is\ P_1 + P_2 = 3VI \cos\phi$$