UNIT II
DC AND AC TRANSIENT ANALYSIS

Introduction

In the preceding lesson, our discussion focused extensively on dc circuits having resistances with either inductor (L) or capacitor (C) (i.e., single storage element) but not both. Dynamic response of such first order system has been studied and discussed in detail. The presence of resistance, inductance, and capacitance in the dc circuit introduces at least a second order differential equation or by two simultaneous coupled linear first order differential equations. We shall see in next section that the complexity of analysis of second order circuits increases significantly when compared with that encountered with first order circuits. Initial conditions for the circuit variables and their derivatives play an important role and this is very crucial to analyze a second order dynamic system.

What is meant by Transients

Whenever a circuit is switched from one condition to another, either by a change in the applied source or a change in the circuit elements, there is a transitional period during which the branch currents and element voltages change from their former values to new ones. This period is called the transient. After the transient has passed, the circuit is said to be in the steady state. Now, the linear differential equation that describes the circuit will have two parts to its solution, the complementary function (or the homogeneous solution) and the particular solution. The complementary function corresponds to the transient, and the particular solution to the steady state.Hence the time-varying currents and voltages resulting from the sudden application of sources, usually due to switching, are called transients. By writing circuit equations, we obtain integro differential equations.

Initial Conditions in Networks:
There are many reasons for studying initial (and final) conditions;

(i) The most important reason is that initial conditions must be known to evaluate the arbitrary constants that appear in the general solution of the differential equations

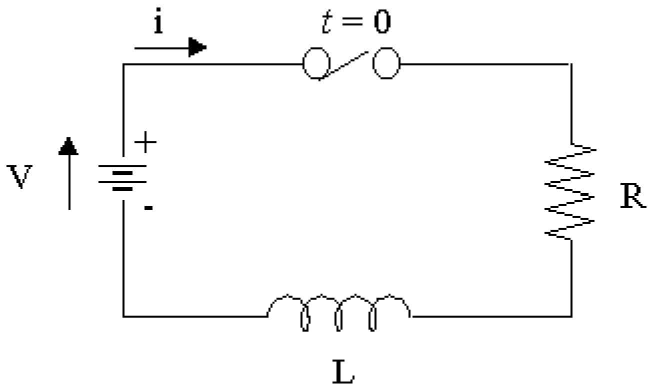
(ii) (ii)The initial conditions give knowledge of the behavior of the elements at the instant of switching At reference time t=0, the switch is closed (we assume that switch act in zero time). To differentiate between the time immediately before and immediately after the operation of a switch, we will use –

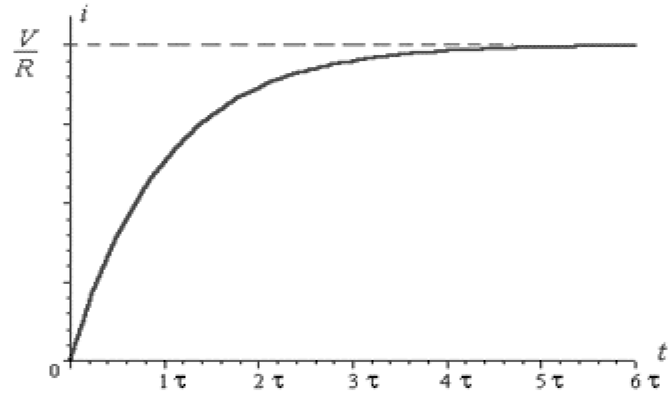
ve and +ve signs. Thus conditions existing just before the switch is operated will be designated as i (0^-), v (0-) etc. Conditions after as i (0+), v (0+) etc. Initial conditions in a network depend on the past history of the network prior to t =0 and the network structure at t = 0+, after switching. The evaluation of all voltages and currents and their derivatives at t =0+, constitutes the evaluation of initial conditions.
Sometimes we use conditions at t =∞; these are known as final conditions.

The steps in determining the forced response for RLC circuits with dc sources are:
1. Replace capacitances with open circuits.
2. Replace inductances with short circuits.
3. Solve the remaining circuit

Series RL Circuit

Consider a circuit in which resistance is connected in series with inductor and voltage source of V volts, s applied across it. Initially the switch is open. Let us say at time 't' we close the switch and the current 'i' starts flowing in the circuit but it does not attains its maximum value rapidly due to the presence of inductor in the circuit as we know inductor has a property to oppose the change in the current flowing through it.





Apply Kirchhoff's voltage law in the above series RL circuit ,

$$V - iR - L\frac{di}{dt} = 0$$

Rearranging the above equation,

$$V - iR = L\frac{di}{dt}$$

$$\frac{dt}{L} = \frac{di}{V - iR}$$

Integrating both sides, we get,

$$\int_0^t \frac{dt}{L} = \int_0^i \frac{di}{V - iR}$$

$$or\ \frac{t}{L} = \int_0^i \frac{di}{V - iR}$$

Now integrate right hand side by using substitution method,

$$Let\ z = V - iR\ or\ \frac{dz}{di} = -R$$

Substituting the values we get,

$$or\ \frac{t}{L} = -\frac{1}{R}\int \frac{dz}{z}$$

We know that integration of,

$$\frac{1}{z} = ln(z)$$

So we get,

$$-\frac{Rt}{L} = ln(V - iR)_0^i$$

By applying limits we get,

$$-\frac{Rt}{L} = ln(V - iR) - ln(V)$$

Simplifying again,

$$-\frac{Rt}{L} = ln\frac{V - iR}{V}$$

Taking antilog on both sides,

$$e^{-\frac{Rt}{L}} = e^{ln\frac{V-iR}{V}}$$

We know that $e^{\ln x} = x$, so we get,

$$e^{-\frac{Rt}{L}} = \frac{V - iR}{V}$$

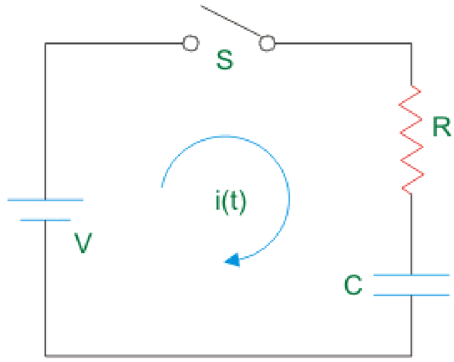
Moving the term containing 'i' on one side we get,

$$i = \frac{V(1 - e^{-\frac{Rt}{L}})}{R}$$

The term L/R in the equation is called the Time Constant, (τ) of the RL series circuit, and it is defined as time taken by the current to reach its maximum steady state value and the term V/R represents the final steady state value of current in the circuit.

Series RC circuit

When a voltage is suddenly applied across a capacitor, which are previously uncharged, electrons shifting from source to capacitor to source start immediately. In other words, accumulation of change in the capacitor starts instantly. As the charge accumulating in the capacitor increases, the voltage developed across the capacitor increases. The voltage developed across the capacitor approaches to supply voltage the rate of charge accumulation in the capacitor decreases accordingly. When these two voltages become equal to each other there will be no more flow of charge from source to capacitor. The flows of electrons from source to capacitor and capacitor to source are nothing but electric current. At the beginning, this current will be maximum and after certain time the current will become zero. The duration in which current changes in capacitor is known as transient period. The phenomenon of charging current or other electrical quantities like voltage, in capacitor is known as transient.



below, Now, if the switch S is suddenly closed, the current starts flowing through the circuit. Let us current at any instant is i(t). Also consider the voltage developed at the capacitor at that instant is V_c(t). Hence, by applying Kirchhoff's Voltage Law, in that circuit we get, $V = Ri(t) + v_c(t) \ldots\ldots (i)$ Now, if transfer of charge during this period (t) is q coulomb, then i(t) can be written as $\frac{dq(t)}{dt}$ Therefore,

$$i(t) = \frac{dq(t)}{dt} \quad \Rightarrow dq(t) = i(t)dt \quad \Rightarrow \int dq(t) = \int i(t)dt$$

$$\Rightarrow \int i(t)dt = q \quad Again,\ q = Cv_c(+)$$

$$\therefore \int i(t)dt = Cv_c(+) \quad \Rightarrow i(t) = C\frac{dv_c(t)}{dt}$$

Putting this expression of i(t) in equation (i) we get,

$$V = RC\frac{dv_c(+)}{dt} + v_c(+) \quad \Rightarrow RC\frac{dv_c(+)}{dt} = V - v_c(+)$$

$$\Rightarrow \frac{dt}{RC} = \frac{dv_c(+)}{V - v_c(+)}$$

Now integrating both sides with respect to time we get, $\frac{t}{RC} = -log\,(V - v_c(t)) + K$ Where, K is a constant can be determined from initial condition. Let us consider the time t = 0 at the instant of switching on the circuit putting t = 0 in above equation we get,

$$-log\,(V - v_c(0)) + K = 0 \Rightarrow\ K + logV$$
$$as,\ v_c(0) = 0$$

There will

be no voltage developed across capacitor at t = 0 as it was previously unchanged. Therefore,

$$t/Rc = -log(V - v_c(t)) + log\ (V) \quad \Rightarrow -t/Rc = log\ [V - v_c(t)] - logV$$

$$\Rightarrow -t/Rc = log\left[\frac{V - v_c(t)}{V}\right] \quad \Rightarrow e^{-t/Rc} = \frac{V - v_c(t)}{V}$$

$$\Rightarrow v_c(t) = V - Ve^{-t/Rc} \quad \Rightarrow v_c(t) = V[1 - e^{-t/Rc}].....(ii)$$

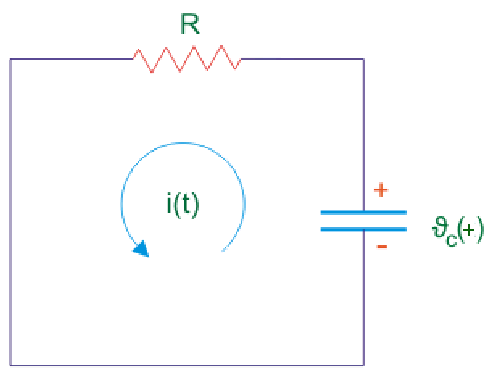
Now if we put RC = t at above equation, we get $V_c = 0.632V$ This RC or product of resistance and capacitance of RC series circuit is known as time constant of the circuit. So, time constant of an RC circuit, is the time for which voltage developed or dropped across the capacitor is 63.2 % of the supply voltage. This definition of time constant only holds good when the capacitor was initially unchanged. Again, at the instant of switching on the circuit i.e. t = 0, there will be no voltage developed across the capacitor. This can also be proved from equation (ii). $v_c(0) = V[1 - e^o] = V[1 - 1] = 0$ So initial current through the circuit is, V/R and let us consider it as I_0. Now at any instant, current through the circuit will be,

$$i(t) = \frac{V - v_c(t)}{R} = \frac{V - V[1 - e^{-t/Rc}]}{R} = \frac{V}{R}e^{-t/Rc} = I_o e^{-t/Rc}$$

Now when, t = Rc the circuit current. $I = I_o e^{-1} = 0.367 I_o$ So at the instant when, current through the capacitor is 36.7 % of the initial current, is also known as time constant of the RC circuit. The time constant is normally denoted will τ (taw). Hence, $\tau = Rc$

Transient During Discharging a Capacitor

Now, suppose the capacitor is fully charged, i.e. voltage at capacitor is equal to the voltage of source. Now if the voltage source is disconnected and instead two terminals of the battery are short circuited, the capacitor will stared discharging means, unequal distribution of electrons between two plates will be equalized through the short circuit path. The process of equaling electrons concentration in two plates will continue until the voltage at capacitor becomes zero. This process is known as discharging of capacitor. Now we will examine the transient behaviorf capacitor during discharging. Now, from the above circuit by applying Kirchhoff Current Law,



$$Ri(t) = -v_c(+)$$

we get, $Now,\ Rc\dfrac{dv_c(+)}{dt} = -v_c(+) \ \Rightarrow -t/Rc = \dfrac{dv_c(+)}{v_c(+)}$ Integrating both sides we get, $-\int dt/Rc = \int \dfrac{dv_c(t)}{v_c(t)} \ \Rightarrow t/Rc = log v_c(t) + K$ K is the constant which can be determined from initial value. Now, at the time of short circuiting the capacitor,

$$t = 0, v_c(0) = V$$

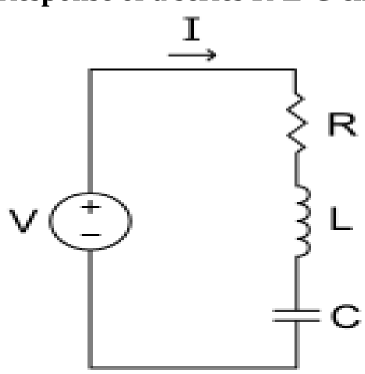
$$\therefore t/Rc = logV + K \ \Rightarrow K = -logV$$

$$\therefore -t/Rc = log v_c(t) - logV \ \Rightarrow -t/Rc = log.\frac{v_c(t)}{V}$$

$$\Rightarrow v_c(t) = Ve^{-t/Rc} \ldots\ldots (iii)$$

Now, from equation (iii), by applying t = τ = RC we get, $v_c(\tau) = V.e^{-1} = 0.368V$ Again, circuit current at that time i.e. τ = RC, $|\iota(\tau)| = \dfrac{v_c(\tau)}{R} = \dfrac{0.368V}{R} = 0.368I_o$ Thus at time constant of capacitor, both capacitor voltage, ϑ_c and current i are reduced to 36.8 % of their initial value

Response of a series R-L-C circuit due to a dc voltage source



In this circuit, the three components are all in series with the voltage source. The governing differential equation can be found by substituting into Kirchhoff's voltage law (KVL) the constitutive equation for each of the three elements. From KVL,

$$v_R + v_L + v_C = v(t)$$

where v_R, v_L, v_C are the voltages across R, L and C respectively and $v(t)$ is the time varying voltage from the source. Substituting in the constitutive equations,

$$Ri(t) + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{\tau=t} i(\tau)\, d\tau = v(t)$$

For the case where the source is an unchanging voltage, differentiating and dividing by L leads to the second order differential equation:

$$\frac{d^2i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{LC}i(t) = 0$$

This can usefully be expressed in a more generally applicable form:

$$\frac{d^2i(t)}{dt^2} + 2\alpha\frac{di(t)}{dt} + {\omega_0}^2 i(t) = 0$$

α and ω_0 are both in units of angular frequency. α is called the neper frequency, or attenuation, and is a measure of how fast thetransient response of the circuit will die away after the stimulus has been removed. Neper occurs in the name because the units can also be considered to be nepers per second, neper being a unit of attenuation. ω_0 is the angular resonance frequency.[3]

For the case of the series RLC circuit these two parameters are given by:[4]

$$\alpha = \frac{R}{2L}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

A useful parameter is the damping factor, ζ, which is defined as the ratio of these two; although, sometimes α is referred to as the damping factor and ζ is not used.[5]

$$\zeta = \frac{\alpha}{\omega_0}$$

In the case of the series RLC circuit, the damping factor is given by,

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}}$$

The value of the damping factor determines the type of transient that the circuit will exhibit.

The differential equation for the circuit solves in three different ways depending on the value of ζ. These are underdamped ($\zeta < 1$), overdamped ($\zeta > 1$) and critically damped ($\zeta = 1$). The differential equation has thecharacteristic equation,

$$s^2 + 2\alpha s + {\omega_0}^2 = 0$$

The roots of the equation in s are,[7]

$$s_1 = -\alpha + \sqrt{\alpha^2 - {\omega_0}^2}$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - {\omega_0}^2}$$

The general solution of the differential equation is an exponential in either root or a linear superposition of both,

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

The coefficients A_1 and A_2 are determined by the boundary conditions of the specific problem being analysed. That is, they are set by the values of the currents and voltages in the circuit at the onset of the transient and the presumed value they will settle to after infinite time.

Overdamped response

The overdamped response ($\zeta > 1$)

$$i(t) = A_1 e^{-\omega_0\left(\zeta+\sqrt{\zeta^2-1}\right)t} + A_2 e^{-\omega_0\left(\zeta-\sqrt{\zeta^2-1}\right)t}$$

The overdamped response is a decay of the transient current without oscillation.

Underdamped response

The underdamped response ($\zeta < 1$)

$$i(t) = B_1 e^{-\alpha t}\cos(\omega_d t) + B_2 e^{-\alpha t}\sin(\omega_d t)$$

By applying standard trigonometric identities the two trigonometric functions may be expressed as a single sinusoid with phase shift,[12]

$$i(t) = B_3 e^{-\alpha t}\sin(\omega_d t + \varphi)$$

The frequency ω_d is given by,

$$\omega_d = \sqrt{\omega_0{}^2 - \alpha^2} = \omega_0\sqrt{1-\zeta^2}$$

This is called the damped resonance frequency or the damped natural frequency. It is the frequency the circuit will naturally oscillate at if not driven by an external source. The resonance frequency, ω_0, which is the frequency at which the circuit will resonate when driven by an external oscillation, may often be referred to as the undamped resonance frequency to distinguish it.

Critically damped response

The critically damped response ($\zeta = 1$) is,[14]

$$i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$$

The critically damped response represents the circuit response that decays in the fastest possible time without going into oscillation. This consideration is important in control systems where it is required to reach the desired state as quickly as possible without overshooting. D_1 and D_2 are arbitrary constants determined by boundary conditions.

Laplace domain

The series RLC can be analyzed for both transient and steady AC state behavior using the Laplace transform.[16] If the voltage source above produces a waveform with Laplace-transformed V(s) (where s is the complex frequency $s = \sigma + i\omega$), KVL can be applied in the Laplace domain:

$$V(s) = I(s)\left(R + Ls + \frac{1}{Cs}\right)$$

Where I (s) is the Laplace-transformed current through all components. Solving for I(s):

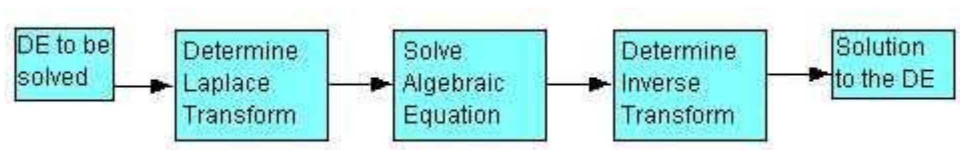
$$I(s) = \frac{1}{R + Ls + \frac{1}{Cs}} V(s)$$

And rearranging, we have that

$$I(s) = \frac{s}{L\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)} V(s)$$

Laplace Transforms

Laplace transformation reduces the problem of solving a dif- ferential equation to an algebraic problem



Definition of Laplace Transform

The Laplace transform is a frequency-domain approach for continuous time signals irrespective of whether the system is stable or unstable. The Laplace transform of a function *f*(*t*), defined for all real numbers $t \geq 0$, is the function *F*(*s*) . where *s* is a complex number frequency parameter

$s = \sigma + i\omega$, with real numbers σ and ω.

Let f(t) be the function of t, time for all t ≥ 0 Then the Laplace transform of f(t), F(s) can be defined as $F(s) = \int_0^{\infty} f(t).e^{-st} dt$ Provided that the integral exists. Where the Laplace Operator, s = σ+jω; will be real or complex j= √(-1) .

Inverse Laplace transform

$$f(t) = \mathcal{L}^{-1}\{F\}(t) = \frac{1}{2\pi i} \lim_{T\to\infty} \int_{\gamma - iT}^{\gamma + iT} e^{st} F(s)\, ds,$$

TABLE OF LAPLACE TRANSFORMS

$f(t)$	$F(s)$
$\delta(t)$	1
$H(t-a)$	$\dfrac{e^{-as}}{s}$
1	$\dfrac{1}{s}$
t^n	$\dfrac{n!}{s^{n+1}}$
e^{kt}	$\dfrac{1}{s-k}$
$t^n e^{kt}$	$\dfrac{n!}{(s-k)^{n+1}}$
$\sin(\omega t)$	$\dfrac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	$\dfrac{s}{s^2+\omega^2}$
$e^{kt}\sin(\omega t)$	$\dfrac{\omega}{(s-k)^2+\omega^2}$
$e^{kt}\cos(\omega t)$	$\dfrac{(s-k)}{(s-k)^2+\omega^2}$
$\sinh(\omega t)$	$\dfrac{\omega}{s^2-\omega^2}$
$\cosh(\omega t)$	$\dfrac{s}{s^2-\omega^2}$
$t\sin(\omega t)$	$\dfrac{2\omega s}{(s^2+\omega^2)^2}$
$t\cos(\omega t)$	$\dfrac{s^2-\omega^2}{(s^2+\omega^2)^2}$

SOME RULES OF LAPLACE TRANSFORMS

$f(t), g(t), h(t)$	$F(s), G(s), H(s)$
$f(t)$	$\int_0^\infty e^{-st} f(t)dt$
$af(t) + bg(t)$	$aF(s) + bG(s)$ (Linearity)
$e^{kt} f(t)$	$F(s-k)$ (Shift in s)
$f'(t)$	$sF(s) - f(0)$
$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \dots f^{(n-1)}(0)$
$\int_0^t f(u)du$	$\frac{1}{s}F(s)$
$H(t-a)f(t-a)$	$e^{-as}F(s)$ (Shift in t)
$f(t+T) = f(t)$	$\frac{1}{1-e^{-sT}}\int_0^T e^{-st} f(t)dt$ (Periodic functions)
$\int_0^t f(t-\tau)g(\tau)d\tau$	$F(s)G(s)$ (Convolution)
$tf(t)$	$-\frac{d}{ds}F(s)$
$\frac{1}{t}f(t)$	$\int_s^\infty F(u)du$

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty}\{sF(s)\}$$
$$\lim_{t\to\infty} f(t) = \lim_{s\to 0}\{sF(s)\}$$

Disadvantages of the Laplace Transformation Method

Laplace transforms can only be used to solve complex differential equations and, it does have a disadvantage, that this method can be used to solve differential equations with known constants.