UNIT V
FILTERS AND FOURIER ANALYSIS OF AC CIRCUITS

PASSIVE FILTERS

Frequency-selective or filter circuits pass to the output only those input signals that are in a desired range of frequencies (called pass band). The amplitude of signals outside this range of frequencies (called stop band) is reduced (ideally reduced to zero). Typically in these circuits, the input and output currents are kept to a small value and as such, the current transfer function is not an important parameter. The main parameter is the voltage transfer function in the frequency domain, Hv (jω) = Vo/Vi . As Hv (jω) is complex number, it has both a magnitude and a phase, filters in general introduce a phase difference between input and output signals. To minimize the number of subscripts, hereafter, we will drop subscript v of Hv. Furthermore, we concentrate on the the "open-loop" transfer functions, Hvo, and denote this simply by H(jω).

Low-Pass Filters

An ideal low-pass filter's transfer function is shown. The frequency between the pass- and-stop bands is called the cut-off frequency (ωc). All of the signals with frequencies below ωc are transmitted and all other signals are stopped.

In practical filters, pass and stop bands are not clearly defined, |H(jω)| varies continuously from its maximum toward zero. The cut-off frequency is, therefore, defined as the frequency at which |H(jω)| is reduced to 1/√2 = 0.7 of its maximum value. This corresponds to signal power being reduced by 1/2 as P ∝ V 2.

Band-pass filters

A band pass filter allows signals with a range of frequencies (pass band) to pass through and attenuates signals with frequencies outside this range.

Band

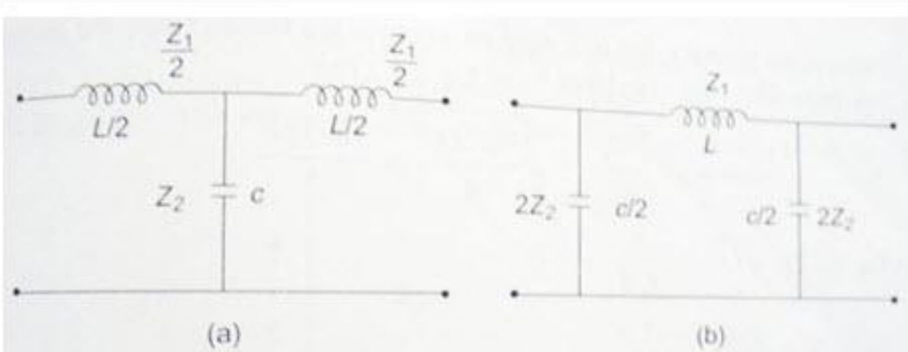
Constant – K Low Pass Filter

A network, either *T* or \[\pi\], is said to be of the constant-*k* type if Z_1 and Z_2 of the network satisfy the relation

$$Z_1Z_2 = k^2 \backslash$$

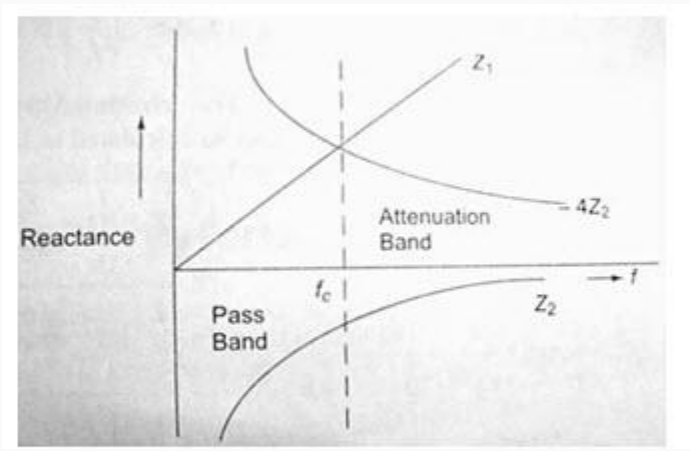
where Z_1 and Z_2 are impedance in the T and \[\pi\] sections as shown in Fig.17.8. Equation 17.20 states that Z_1 and Z_2 are inverse if their product is a constant, independent of frequency. *k* is a real constant, that is the resistance. *k* is often termed as design impedance or nominal impedance of the constant *k*-filter.

The constant *k, T* or \[\pi\] type filter is also known as the prototype because other more complex networks can be derived from it.



where $Z_1 = j\omega_L$ and $Z_2 = 1/j\omega_C$. Hence $Z_1Z_2 = {L \over C}={k^2}$ which is independent of frequency

The pass band can be determined graphically. The reactances of Z_1 and $4Z_2$ will vary with frequency as drawn in Fig.30.2. The cut-off frequency at the intersection of the curves Z_1 and $4Z_2$ is indicated as f_c. On the X-axis as $Z_1 = -4Z_2$ at cut-off frequency, the pass band lies between the frequencies at which $Z_1 = 0$, and $Z_1=-4Z_2$.



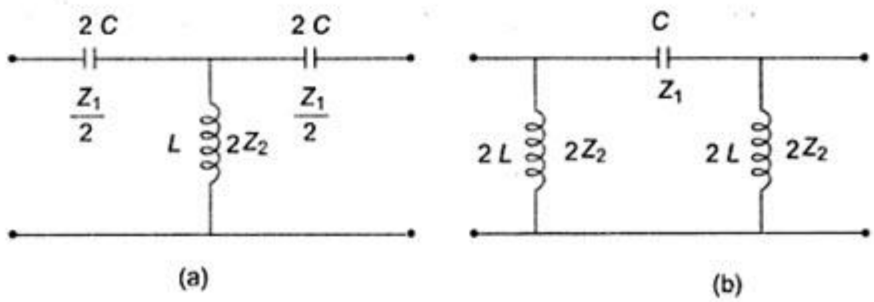
All the frequencies above f_c lie in a stop or attenuation band

The characteristic impedance of a π-network is given by

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{k}{\sqrt{1-\left(\frac{f}{f_c}\right)^2}} \qquad \ldots\ldots (30.5)$$

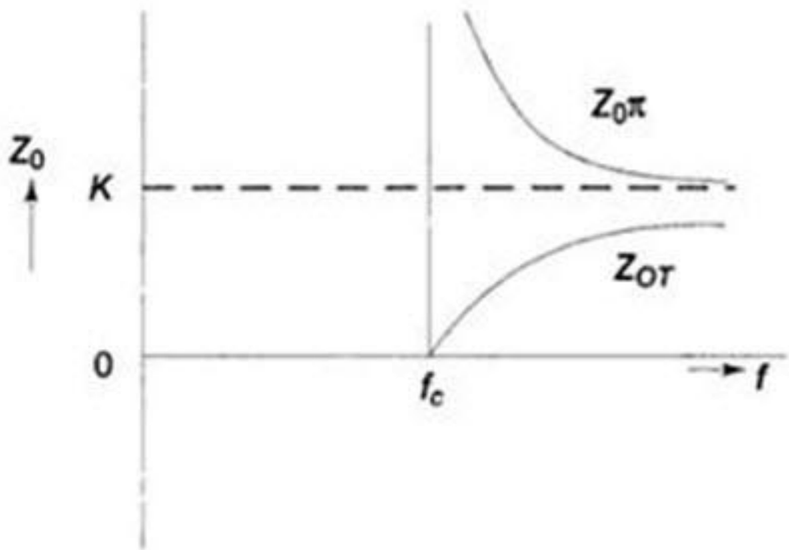
Constant K-High Pass Filter

Constant K-high pass filter can be obtained by changing the positions of series and shunt arms of the networks shown in Fig.30.1. The prototype high pass filters are shown in Fig.30.5, where $Z_1 = -j/\omega_C$ and $Z_2 = j\omega L$.



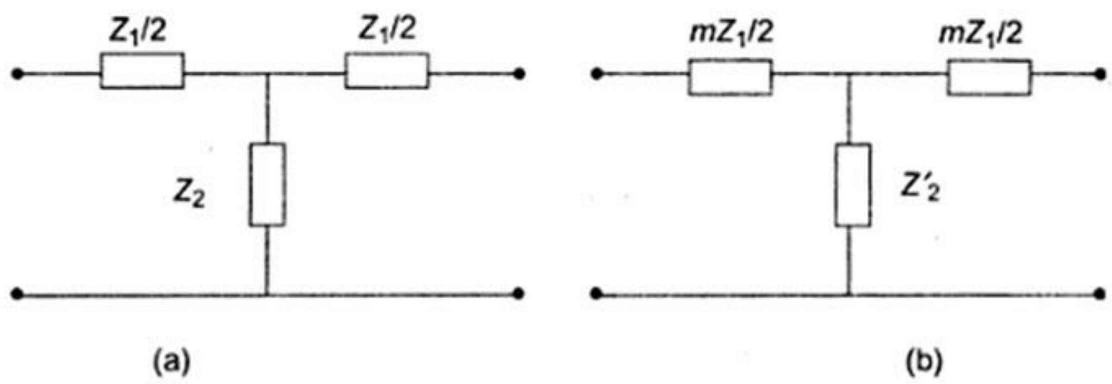
Again, it can be observed that the product of Z_1 and Z_2 is independent of frequency, and the filter design obtained will be of the constant k type.

The plot of characteristic impedance with respect to frequency is shown



m-Derived T-Section

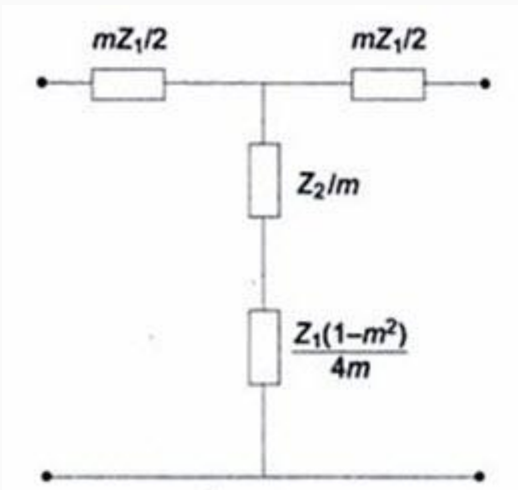
It is clear from previous chapter Figs 30.3 & 30.7 that the attenuation is not sharp in the stop band for k-type filters. The characteristic impedance, Z_0 is a function of frequency and varies widely in the transmission band. Attenuation can be increased in the stop band by using ladder section, i.e. by connecting two or more identical sections. In order to join the filter sections, it would be necessary that their characteristic impedance be equal to each other at all frequencies. If their characteristic impedances match at all frequencies, they would also have the same pass band. However, cascading is not a proper solution from a practical point of view. This is because practical elements have a certain resistance, which gives rise to attenuation in the pass band also. Therefore, any attempt to increase attenuation in stop band by cascading also results in an increase of 'a' in the pass band. If the constant k section is regarded as the prototype, it is possible to design a filter to have rapid attenuation in the stop band, and the same characteristic impedance as the prototype at all frequencies. Such a filter is called m-derived filter. Suppose a prototype T-network shown in Fig.31.1 (a) has the series arm modified as shown in Fig.31.1 (b), where m is a constant. Equating the characteristic impedance of the networks in we have



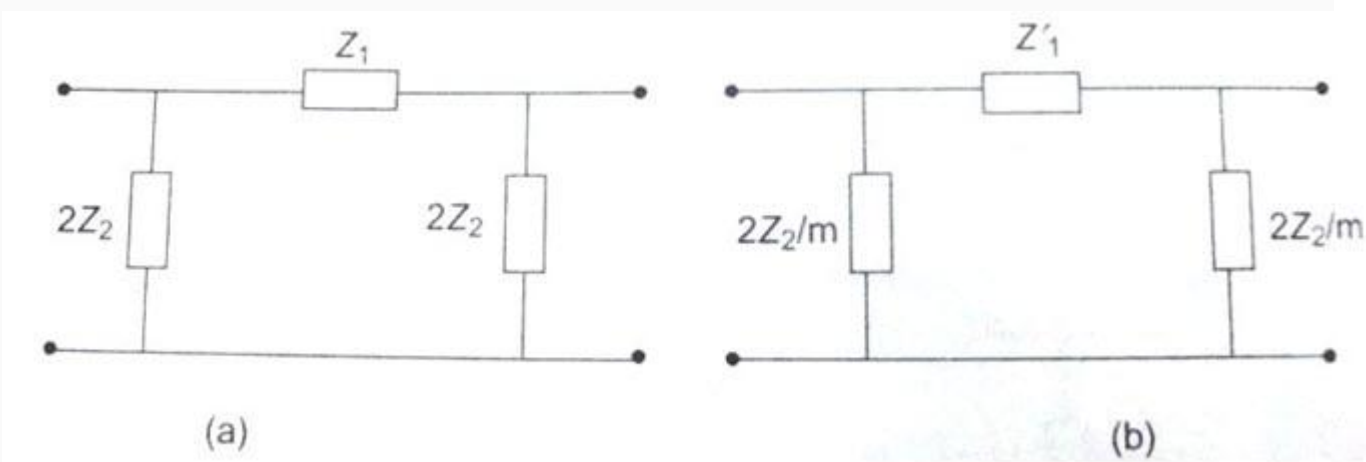
$Z_{0T} = Z_{0T'}$

where $Z_{0T'}$ is the characteristic impedance of the modified (m-derived) T-network.

Thus m-derived section can be obtained from the prototype by modifying its series and shunt arms. The same technique can be applied to \[\pi\] section network. Suppose a prototype p-network shown in Fig.31.3 (a) has the shunt arm modified as shown in Fig.31.3 (b).



The characteristic impedances of the prototype and its modified sections have to be equal for matching.



the characteristic impedance of the modified (m-derived) \[\pi\]-network

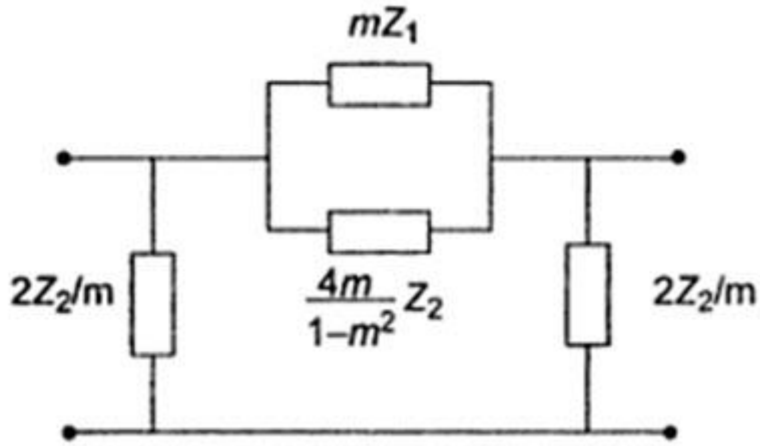
$$\therefore \quad \sqrt{\frac{Z_1 Z_2}{1+\frac{Z_1}{4Z_2}}} = \sqrt{\frac{Z_1^{'} \frac{Z_2}{m}}{1+\frac{Z_1^{'}}{4.Z_2/m}}}$$

Or

$$Z_1^{'} = \frac{Z_1 Z_2}{\frac{Z_1}{4m}+\frac{Z_2}{m}-\frac{mZ_1}{4}}$$

$$= \frac{Z_1 Z_2}{\frac{Z_2}{m}+\frac{Z_1}{4m}\left(1-m^2\right)}$$

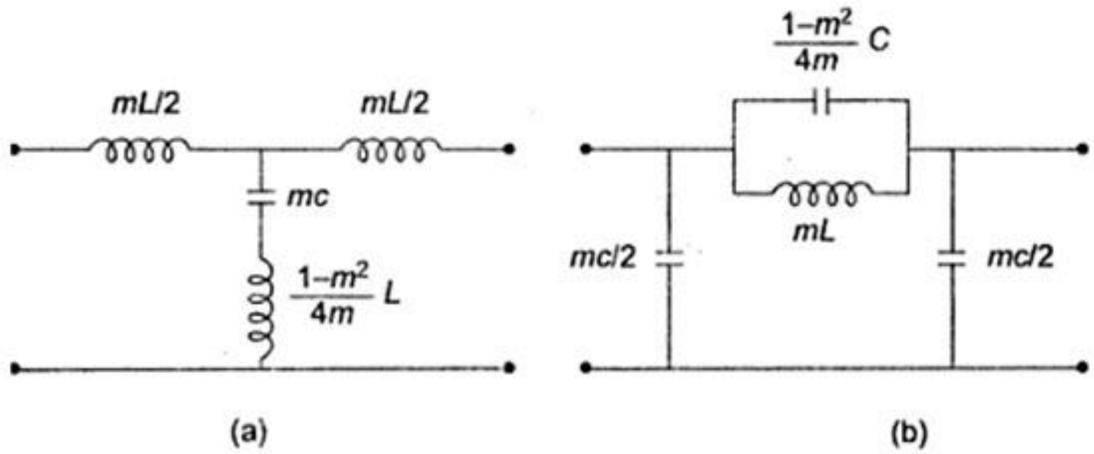
$$Z_1^{'} = \frac{Z_1 Z_2 \frac{4m^2}{\left(1-m^2\right)}}{\frac{Z_2 4m_2}{m\left(1-m_2\right)}+Z_1 m} = \frac{mZ_1 \frac{Z_2 4m}{\left(1-m^2\right)}}{mZ_1+\frac{Z_2 4m}{\left(1-m^2\right)}} \qquad (31.2)$$



the series arm of the m-derived \[\pi\] section is a parallel combination of mZ_1 and $4mZ_2/1-m^2$

m-Derived Low Pass Filter

In Fig.31.5, both m-derived low pass T and \[\pi\] filter sections are shown. For the T-section shown Fig.31.5 (a), the shunt arm is to be chosen so that it is resonant at some frequency f_x above cut-off frequency f_c its impedance will be minimum or zero. Therefore, the output is zero and will correspond to infinite attenuation at this particular frequency.



$$m\omega_r L = \frac{1}{\left(\frac{1-m^2}{4M}\right)\omega_r C}$$

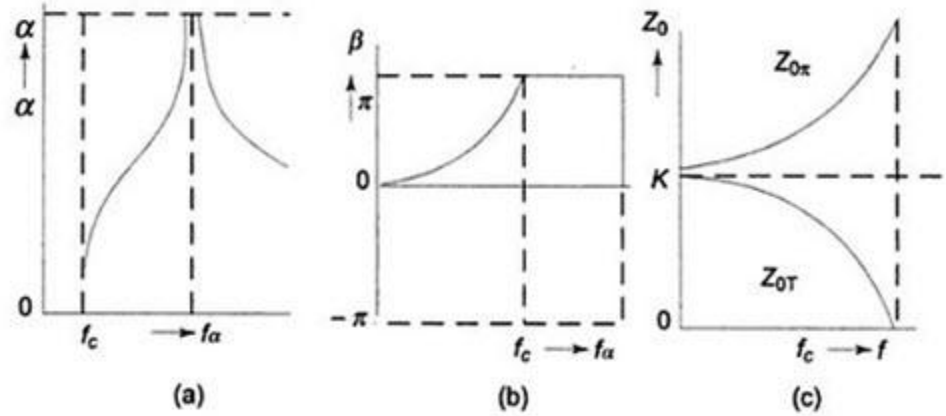
$$\omega_r^2 = \frac{4}{LC(1-m^2)}$$

$$f_r = \frac{1}{\pi\sqrt{LC(1-m^2)}}$$

The variation of attenuation for a low pass m-derived section can be verified

$$\therefore \qquad \alpha = 2\cosh^{-1}\frac{m\frac{f}{f_c}}{\sqrt{1-\left(\frac{f}{f_\alpha}\right)^2}}$$

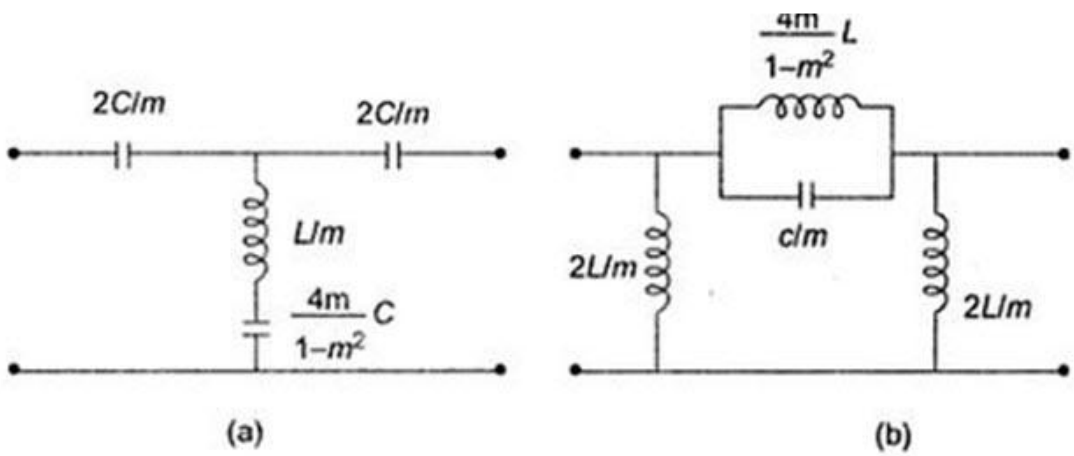
$$\text{And} \qquad \beta = 2\sin^{-1}\sqrt{\left|\frac{Z_1}{4Z_1}\right|} = 2\sin^{-1}\frac{m\frac{f}{f_c}}{\sqrt{1-\left(\frac{f}{f_c}\right)^2(1=m)^2}}$$



M-derived High Pass Filter

If the shunt arm in T-section is series resonant, it offers minimum or zero impedance. Therefore, the output is zero and, thus, at resonance frequency, or the frequency corresponds to infinite attenuation.

$$\omega_r \frac{L}{m} = \frac{1}{\omega_r \frac{4m}{1-m^2} C}$$

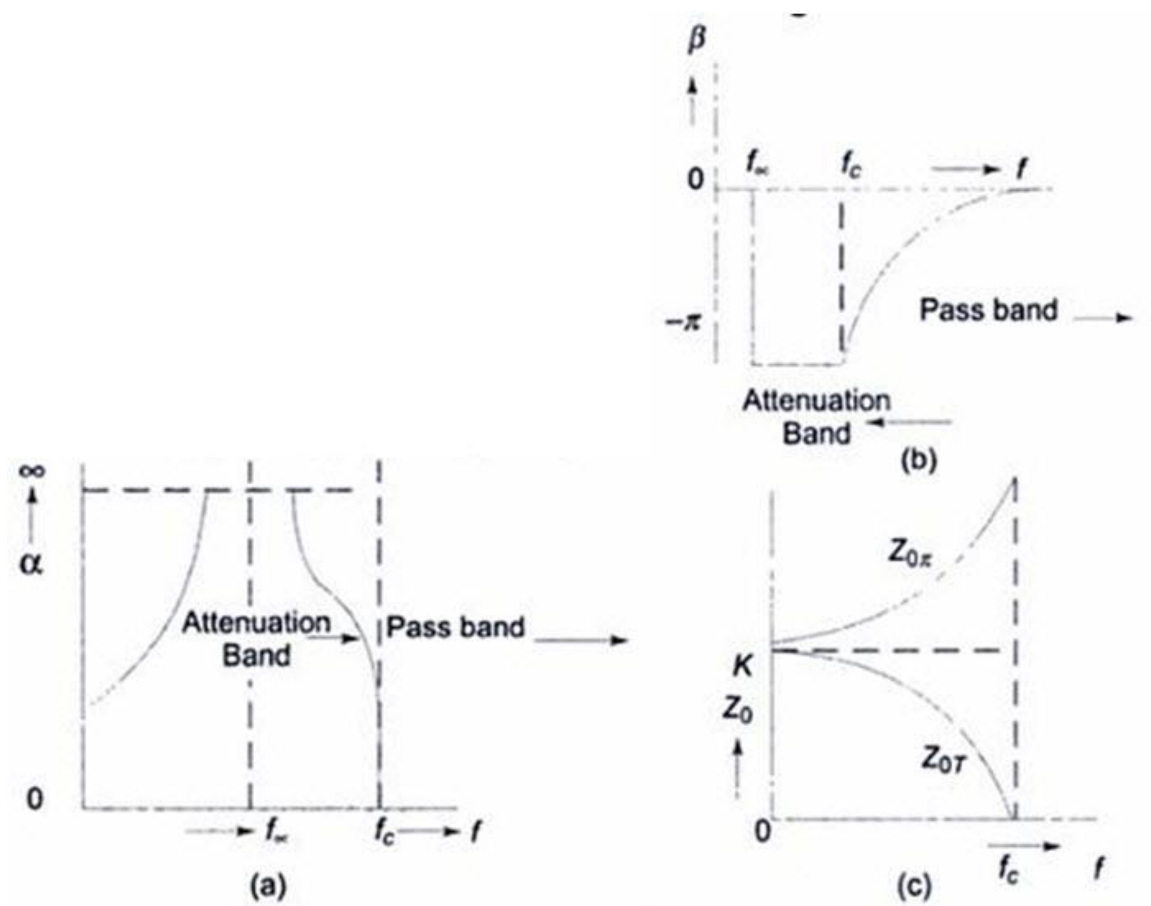


the m-derived \[\pi\]-section, the resonant circuit is constituted by the series arm inductance and capacitance.

$$\frac{4m}{1-m^2}\omega_r L = \frac{1}{\frac{\omega_r}{m}C}$$

$$\omega_r^2 = \omega_\alpha^2 = \frac{1-m^2}{4LC}$$

$$\omega_\alpha = \frac{\sqrt{1-m^2}}{2\sqrt{LC}} \; or \; f_\alpha = \frac{\sqrt{1-m^2}}{4\pi\sqrt{LC}}$$



Fourier Series and Fourier Integrals

The type of series that can represent a a much larger class of functions is called Fourier Series. These series have the form

$$g(t) = a_0 + \sum_{m=1}^{\infty} a_m \cos\left(\frac{2\pi mt}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nt}{T}\right)$$

$$= \sum_{m=0}^{\infty} a_m \cos\left(\frac{2\pi mt}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2\pi nt}{T}\right)$$

$$a_0 = \frac{1}{T}\int_0^T f(t)dt$$

$$a_m = \frac{2}{T}\int_0^T f(t)\cos\left(\frac{2\pi mt}{T}\right)dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin\left(\frac{2\pi nt}{T}\right)dt$$

The coefficients an and bn are called Fourier coefficients. this series represents a periodic function with period T. To represent function f(x) in this way, the function has to be (1) periodic with just a finite number of maxima and minima within one period and just a finite number of discontinuities, (2) the integral over one period of |f(x)| must converge. If these conditions are satisfied and one period of f(x) is given on an interval (x0, x0 + T), the Fourier coefficients an and bn can be computed using the above formulae.

Fourier Transformation

The Fourier transform is an integral operator meaning that it is defined via an integral and that it maps one function to the other. If you took a differential equations course, you may recall that the Laplace transform is another integral operator you may have encountered. The Fourier transform represents a generalization of the Fourier series. Recall that the Fourier series is f(t) = P∞ n=−∞ cne 2nπit T . The sequence cn can be regarded as a function of n and is called Fourier spectrum of f(t). We can think of c(n) being another representation of f(t), meaning that f(t) and c(n) are different representations of the same object. Indeed: given f(t) the coefficients c(n) can be computed and, conversely, given c(n), the Fourier series with coefficients c(n) defines a function f(t). We can plot c(n) as a function of n (and get a set of infinitely many equally spaced points). In this case we think of c as a function of n, the wave number. We can also think of c as a function of ω = 2nπ T , the frequency. Note that if T is large, then ω is small and the function T cn becomes a continuous function of ω for T → ∞. Note also that if we let T → ∞, the requirement that f(t) is periodic can be waved since the period becomes (−∞,∞). The Fourier Transform F(ω) of f(t) is the limit of the continuous function √ 1 2π T cn when T → ∞.

Linearity

The F.T. is linear:

$$\mathcal{F}[af(t) + bg(t)] = a\mathcal{F}[f(t)] + b\mathcal{F}[g(t)]$$

Time/Frequency Duality

The duality property is one that is not shared by the Laplace transform. While slightly confusing perhaps at first, it essentially doubles the size of our F.T. table. The duality property follows from the similarity of the forward and inverse F.T. It states that if

$$f(t) \Leftrightarrow F(\omega)$$

then

$$F(t) \Leftrightarrow 2\pi f(-\omega)$$

where the function on the left is the function of time and the function on the right is the function of frequency.

Scaling Property

We have seen this for Laplace transforms: If

$$f(t) \Leftrightarrow F(\omega)$$

then

$$f(at) \Leftrightarrow \frac{1}{|a|}F(\omega/a)$$

Time-Shift Property

If

$$f(t) \Leftrightarrow F(\omega)$$

then

$$f(t - t_0) \Leftrightarrow F(\omega)e^{-j\omega t_0}$$

Frequency-shift Property

This innocuous-looking property forms a basis for every radio and TV transmitter in the world! It simply states that if

$$f(t) \Leftrightarrow F(\omega)$$

then

$$f(t)e^{j\omega_0 t} \Leftrightarrow F(\omega - \omega_0)$$

Convolution Property

If

$$f_1(t) \Leftrightarrow F_1(\omega) \quad \text{and} \quad f_2(t) \Leftrightarrow F_2(\omega)$$

then

$$f_1(t) * f_2(t) \Leftrightarrow F_1(\omega)F_2(\omega)$$

(where $*$ is convolution) and

$$f_1(t)f_2(t) \Leftrightarrow \frac{1}{2\pi}F_1(\omega) * F_2(\omega)$$

Time Differentiation

If

$$f(t) \Leftrightarrow F(\omega)$$

then

$$\frac{df}{dt} \Leftrightarrow j\omega F(\omega)$$

Time Integration

We saw above that

$$\int_{-\infty}^{t} f(x)dx \Leftrightarrow \frac{F(\omega)}{j\omega} + \pi F(0)\delta(\omega)$$

If f is zero mean (i.e. $F(0) = 0$) then

$$\int_{-\infty}^{t} f(x)dx \Leftrightarrow \frac{F(\omega)}{j\omega}$$

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2. Network Theory : A. Sudhakar Shyammohan Palli